STRUCTURE OF SWITCH-ON SHOCK WAVES

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Shock waves propagating along an undisturbed magnetic field at frequencies $\omega \leqslant \Omega_H \equiv eH_0/m_ic$ ("switch-on" waves) have been studied experimentally in a number of works [1-3]; some of the experiments [1] have been conducted under conditions in which the dissipation mechanism involves Coulomb collisions, while others [2-3] have been conducted under conditions in which this mechanism involves collective interactions. The magnetohydrodynamic structure of switch-on shock waves has been discussed [4] with viscosity $\mu \backsim T^{5/2}$ and finite conductivity $\sigma \backsim T^{3/2}$ taken into account.

In the current study a double-fluid gasdynamic model of a quasineutral plasma is used to study nonstationary switch-on shock waves in order to take into account finite conductivity and electron heat conduction, since it is precisely these dissipative processes that determine the structure of collision-free shock waves [2]. The initial system of equations has the form

$$\begin{gathered}
\partial N/\partial t + \partial(Nu)/\partial x = 0;\\
Nm_i(\partial u/\partial t + u\partial u/\partial x) = -\frac{\partial}{\partial x}\left[p + (H_y^2 + H_z^2)/8\pi\right];\\
Nm_i(\partial v/\partial t + u\partial v/\partial x) = (H_0/4\pi)\partial H_y/\partial x;\\
Nm_i(\partial w/\partial t + u\partial w/\partial x) = (H_0/4\pi)\partial H_z/\partial x;\\
\frac{\partial H_y}{\partial t} = -\frac{\partial}{\partial x}\left\{uH_y - vH_0 + \frac{m_ec^2}{4\pi e^2}\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\left(\frac{1}{N}\frac{\partial H_y}{\partial x}\right) + \frac{c^2}{4\pi\sigma}\frac{\partial H_y}{\partial x} + \frac{cH_0}{4\pi eN}\frac{\partial H_z}{\partial x}\right\};\\
\frac{\partial H_z}{\partial t} = \frac{\partial}{\partial x}\left\{wH_0 - uH_z + \frac{m_ec^2}{4\pi e^2}\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)\left(\frac{1}{N}\frac{\partial H_z}{\partial x}\right) + \frac{c^2}{4\pi\sigma}\frac{\partial H_z}{\partial x} - \frac{cH_0}{4\pi eN}\frac{\partial H_y}{\partial x}\right\};\\
\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial x} + \gamma p\frac{\partial u}{\partial x} = (\gamma - 1)\left\{\frac{c^2}{16\pi^2\sigma}\left[\left(\frac{\partial H_y}{\partial x}\right)^2 + \left(\frac{\partial H_z}{\partial x}\right)^2\right] + \frac{\partial}{\partial x}\left(\varkappa\frac{\partial T}{\partial x}\right)\right\};\ p = NT.
\end{gathered}\tag{1}$$

The direction of wave propagation and of the undisturbed magnetic field $\mathbf{H}_0$ coincides with the x axis; $U = \{u, v, w\}$ is the macroscopic plasma velocity; p is electron pressure (the ions are assumed cold); $\sigma = Ne^2/m_e\nu$ is conductivity; ν is the effective plasma particle–electromagnetic field fluctuation frequency; and $\varkappa$ is electron heat conduction.

At the initial moment of time a homogeneous quiescent plasma with $p_0 \ll H_0^2/8\pi$ occupies the region $x \geq 0$; at the plasma–vacuum boundary we define the z component of the magnetic field by the law

$$H_z(0, t) = H_z^0(1 - e^{-\omega t}).$$

Let us consider the results obtained from a numerical solution of the system (1). Figure 1 depicts the spatial distribution of the transverse magnetic field components in a switch-on shock wave obtained as a result of a numerical solution of the system (1) for $\nu = 10^{-2}\omega_H$, $\omega = 0.9\Omega_H$,

$$H_z^0 = 1.2\,H_0,\ \mathrm{M} \simeq 1.2,\ H_0 = 500\,\mathrm{Oe},\ N_0 = 10^{14}\ \mathrm{cm}^{-3}.$$

It is known that two waves whose law of dispersion has the form

$$\begin{gathered}
\omega/k \simeq V_A\sqrt{1 - kc/\Omega_i} \text{ — ordinary wave,}\\
\omega/k \simeq V_A\sqrt{1 + kc/\Omega_i} \text{ — extraordinary wave,}
\end{gathered}\tag{2}$$

propagate along an undisturbed magnetic field in the frequency range $\omega \leq \Omega_H$.

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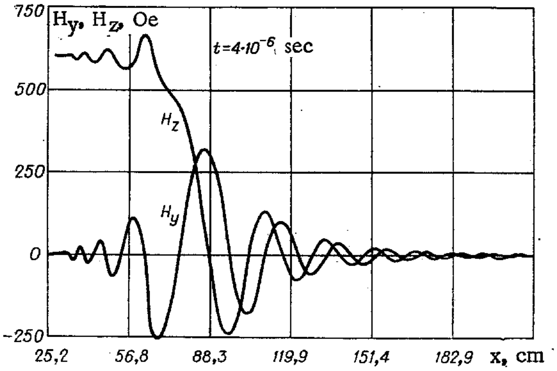

Fig. 1

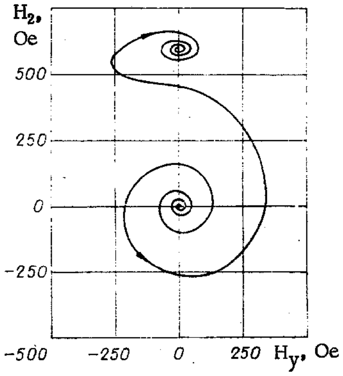

Fig. 2

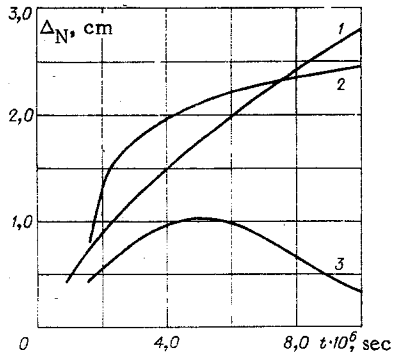

Fig. 3

In accordance with the dispersion law (2), the profile of the z component of the magnetic field has an oscillatory structure both in front of the fundamental shock and behind it. The characteristic spatial scale of the oscillations is on the order of the dispersion distance c/Ω_i. The maximal dimension of the oscillations behind the front $\simeq 4c/\Omega_i$, and in front of it $\simeq 7c/\Omega_i$. The y component of the magnetic field appearing in the front also has an oscillatory structure. The phase shift between H_y and H_z amounts to 90°. The direction of rotation of the transverse magnetic field vector $H_\perp$ in front of the fundamental shock coincides with the direction of rotation of an electron in the magnetic field H_0 and, behind the shock, with the direction of rotation of an ion (Fig. 2).

Thus, in the case of slight dissipation the shock wave has a "corkscrew" structure; the width of the fundamental density shock and the number of oscillations increase over the course of time (Fig. 3, curve 1), though the wave velocity $V = V_A M$ and width of the magnetic field shock in the resulting wave remain roughly constant. The widths of the fundamental density shock and of the magnetic field are determined by the equations

$$\Delta_N = (N_{\max} - N_{\min})/|\partial N/\partial x|_{\max}, \quad \Delta_H = (H_{\max} - H_{\min})/|\partial H/\partial x|_{\max}.$$

An increase in dissipation leads to disappearance of the oscillations, and the dimension of the latter is less than the dissipation distance,

$$\Delta_d \approx c^2/4\pi\sigma V_A(M - 1).$$

A switch-on shock wave is quasistationary when $\nu = 0.6\ \omega_H$ and $M \simeq 1.2$ and for an amplitude of the magnetic field at the plasma boundary of $H_z^0 = 1.2H_0$; the velocity and width of the field shock and wave density remain practically invariant over the course of time (Fig. 3, curve 2). An analysis of the equations for the structure of stationary switch-on waves shows that when $M > M_* = 1.53$, without heat conduction, and $M > M_* = 1.63$, with heat conduction, the gasdynamic functions N and u undergo a discontinuity in the case of a continuous magnetic field. By solving the nonstationary equations (1) for high field amplitudes at the boundary, it becomes possible to trace the variation of the switch-on wave structure and to approach the flip-flop state, i.e., $\Delta_N \to 0$ (Fig. 3, curve 3).

A comparison of results of our calculation with previous experiments [2] shows that there is qualitative agreement in terms of the structure and velocity of switch-on waves when $M < M_*$. It is difficult to perform a detailed comparison, since no uniform pattern can be seen in the experiments. In addition, regimes with $M \gtrsim M_*$ studied in the experiments were not discussed in the present work.

LITERATURE CITED

1. L. Bighel, A. R. Collins, and R. C. Cross, "MHD 'switch-on' structure," Phys. Lett., A47, No. 4 (1974).
2. R. Kh. Kurtmullaev, V. L. Masalov, K. I. Mekler, and V. N. Semenov, "Shock waves propagating along a magnetic field in a collision-free plasma," Zh. Éksp. Teor. Fiz., 60, No. 1 (1971).
3. K. Kuriki and M. Inutake, "Super-Alfvénic flow and collision-free shock wave in a plasma wind tunnel," Phys. Fluids, 17, No. 1 (1974).
4. R. J. Bickerton, L. Lenamon, and R. V. W. Murphy, "The structure of hydromagnetic shock waves," J. Plasma Phys., 5, No. 2 (1971).

STATISTICAL – PHENOMENOLOGICAL APPROACH TO THE DESCRIPTION OF TURBULENT FLAMES

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INTRODUCTION

Development of a theory for the combustion of gaseous mixtures in a turbulent stream is usually associated with the modeling of the process on the basis of certain particular assumptions [1]. At the same time, in the hydrodynamics of nonreacting flows a method for the statistical averaging of the Navier – Stokes equations is being successfully developed. The resulting correlation moments of the second-order hydrodynamic quantities are not directly related to the average flow parameters; for these it is necessary to set up equations which are similar in structure to conservation laws. Although the higher-order moments which then appear again require the use of phenomenological hypotheses, the chain of two-moment equations constructed in this way does describe a number of qualitatively new effects. The fundamentals of this approach were laid down by A. N. Kolmogorov and have now been widely developed by Soviet and other scientists (see, for example, [2] and the references cited therein): Solutions have been constructed with a small number of empirical constants to give satisfactory qualitative description of a turbulent flow. A similar approach to this is used in [3] to describe a chemical reaction with a linear source of heat (concentration) and in [4] for thermal transport problems. The effect of nonlinear heat production with allowance for pulsations in temperature and concentration in the zeroth-order statement of the problem is studied in [5]; the application to a small-scale turbulent flame is considered in [6-9]. In these last papers, the flame-propagation equation is closed by using the phenomenology of the Prandtl displacement path; a method of averaging the nonlinear thermal production function is proposed. In [10] the equation for the turbulent energy balance is used to study the level of turbulence in a flame where the Prandtl hypothesis is also used for the temperature pulsations. In the present paper, the statistical – phenomenological method is extended to the problem of the turbulent combustion of mixed gases. The principal emphasis is on studying the effect of the chemical reaction on the thermal exchange in the flame and the reverse effect of the turbulence on the reaction rate and hence on the turbulent combustion rate. Approximate estimates of these effects are given.

§1. The presentation given below is based on the following simplifying assumptions: 1) the hydrodynamic field of the averaged $\langle u_i(x_i, t)\rangle$, $\langle p(x_i, t)\rangle$ and pulsating $u_i'(x_i, t)$, $p'(x_i, t)$ motions is known; 2) the medium is incompressible, ρ =const; 3) a single-stage exothermic reaction takes place in the flow in accordance with the equation $nA \to B$ at a rate $\Phi(T, \eta)$ and with calorific value $Q > 0$ (n is the order of the reaction; A is the initial substance; B are the reaction products; T is the temperature; and η is the relative concentration of the reaction products or the degree of combustion); 4) the molecular transport coefficients are independent of the flow parameters and they are subject to the equations $\nu = \varkappa = D_{AB}$ (ν is the viscosity; $\varkappa$ is the thermal conductivity; and D is the diffusion); 5) we consider the usual slow combustion conditions when we can neglect the pressure gradient and the viscous dissipation heat in the flame in comparison with the heat developed by the chemical reaction; and 6) the back reaction of the flame on the hydrodynamics of the flow $\langle u_i\rangle$, u_i', p is not taken into ac-

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